CHAPTER 7: WORK, ENERGY, AND ENERGY RESOURCES

7.1 WORK: THE SCIENTIFIC DEFINITION

1. *How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.*

Solution
$$W = Fd\cos\theta = (5.00\,\text{N})(0.600\,\text{m})\cos 0° = \underline{3.00\,\text{J}}$$
$$W = 3.00\,\text{J}\times\frac{1\,\text{kcal}}{4186\,\text{J}} = \underline{7.17\times10^{-4}\,\text{kcal}}$$

2. *A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task. (Neglect friction in your calculations.)*

Solution
$$W = mgh = (75.0\ \text{kg})\ (9.80\ \text{m/s}^2)\,(2.50\ \text{m}) = 1838\ \text{J} = \underline{1.84\times10^3\ \text{J}}$$

3. *(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?*

Solution
(a)
$$W_\text{c} = Fd\cos\theta = (mg + F_\text{f})\times d$$
$$= \left[(1500\,\text{kg})(9.80\,\text{m/s}^2) + 100\,\text{N}\right]\times 40.0\,\text{m} = \underline{5.92\times10^5\ \text{J}}$$

(b) $W_\text{g} = -mgh = (1500\ \text{kg})\,(-9.80\ \text{m/s}^2)\,(40.0\ \text{m}) = \underline{-5.88\times10^5\ \text{J}}$

(c) The net force is zero, since the elevator moves at a constant speed. Therefore, the total work done is $0\ \text{J}$.

4. *Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See Table 7.1 for the energy content of gasoline.) (a) What is the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?*

Solution $W_{\mathrm{f}} = (2.0\ \mathrm{gal})(1.2\times10^{8}\ \mathrm{J/gal})(0.30) = 7.2\times10^{7}\ \mathrm{J}$

(a) $W_{\mathrm{f}} = F_{\mathrm{f}}d \Rightarrow F_{\mathrm{f}} = \dfrac{W_{\mathrm{f}}}{d} = \dfrac{7.2\times10^{7}\ \mathrm{J}}{1.08\times10^{5}\ \mathrm{m}} = 666.7\ \mathrm{N} = \underline{6.7\times10^{2}\ \mathrm{N}}$

(b) $\dfrac{28.0\ \mathrm{m/s}}{30.0\ \mathrm{m/s}}\times 2.0\ \mathrm{gal} = 1.867\ \mathrm{gal} = \underline{1.9\ \mathrm{gal}}$

5. *Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See Figure 7.32.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate and on his body to get up the ramp.*

Solution $W = Fd = (mg\sin\theta + 500\ \mathrm{N})\times d$

$= \left[(85.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(\sin 20^\circ) + 500\ \mathrm{N}\right]\times 4.00\ \mathrm{m} = \underline{3.14\times10^{3}\ \mathrm{J}}$

6. *How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure 7.33? Assume no friction acts on the wagon.*

Solution

$W = Fd\cos\theta = 50.0\ \mathrm{N}\times 30.0\ \mathrm{m}\times\cos 30^\circ = 1.299\times10^{3}\ \mathrm{J} = \underline{1.30\times10^{3}\ \mathrm{J}}$

7. *A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?*

Solution

(a) $W_{\mathrm{f}} = Fd\cos\theta = 35.0\ \mathrm{N}\times 20.0\ \mathrm{m}\times\cos 180^\circ = \underline{-700\ \mathrm{J}}$

(b) $W_{\mathrm{g}} = Fd\cos\theta = 35.0\ \mathrm{N}\times 20.0\ \mathrm{m}\times\cos 90^\circ = \underline{0\ \mathrm{J}}$

(c) net $W = W_{\mathrm{s}} + W_{\mathrm{f}} = 0$, or $W_{\mathrm{s}} = \underline{700\ \mathrm{J}}$

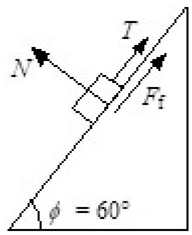
(d) $W_{\mathrm{s}} = Fd\cos\theta$ where $\theta = 25^\circ$, so that:

$$F = \frac{W_{\mathrm{s}}}{d\cos\theta} = \frac{700\ \mathrm{J}}{20.0\ \mathrm{m}\times\cos 25^\circ} = 38.62\ \mathrm{N} = \underline{38.6\ \mathrm{N}}$$

(e) net $W = W_{\mathrm{f}} + W_{\mathrm{s}} = -700\ \mathrm{J} + 700\ \mathrm{J} = \underline{0\ \mathrm{J}}$

8. *Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a $60.0°$ slope at constant speed, as shown in Figure 7.34 The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?*

Solution



$$W_f = F_f d\cos\theta;\ F_f = \mu_s N = \mu_s mg\cos\varphi,$$
$$W_f = \mu_s mg\ \cos\varphi\ (d\cos\theta)$$
$$= (0.100)(90.0\text{ kg})(9.80\text{ m/s}^2)(\cos 60.0°)(30.0\text{ m})(\cos 180°)$$

(a) $$W_f = -1.323\times10^3\text{ J} = \underline{-1320\text{ J}}$$

$$W_r = Td\cos\theta;\ T + F_f = mg\sin\phi$$
$$T = mg\sin\phi - \mu_s mg\cos\phi = mg(\sin\phi - \mu_s\cos\phi)$$
$$W_r = mg(\sin\phi - \mu_s\cos\phi)d\cos\theta$$
$$W_r = (90.0\text{ kg})(9.80\text{ m/s}^2)[\sin 60.0° - (0.100)\cos 60.0°](30.0\text{ m})(\cos 180°)$$

(b) $$= -2.1592\times10^4\text{ J} = \underline{-2.16\times10^4\text{ J}}$$

$$W_g = F_g d\cos\theta = mg\sin\phi\,(d\cos\theta)$$
$$W_g = (90.0\text{ kg})(9.80\text{ m/s}^2)(\sin 60.0°)(30.0\text{ m})(\cos 0°)$$

(c) $$= 2.2915\times10^4\text{ J} = \underline{2.29\times10^4\text{ J}}$$

(d) $$\text{net } W = W_f + W_r + W_g = -1.323\times10^3\text{ J} - 2.1592\times10^4\text{ J} + 2.2915\times10^4\text{ J} = \underline{0\text{ J}}$$

(Since the sled moves at constant speed, this must be so.)

7.2 KINETIC ENERGY AND THE WORK-ENERGY THEOREM

9. *Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.*

Solution
$$27500\text{ km/h}\times\frac{1000\text{ m}}{1\text{ km}}\times\frac{1\text{ h}}{3600\text{ s}}=7.638\times10^{3}\text{ m/s}$$
$$110\text{ km/h}\times\frac{1000\text{ m}}{1\text{ km}}\times\frac{1\text{ h}}{3600\text{ s}}=30.56\text{ m/s}$$
$$\text{KE}_{\text{tr}}=\frac{1}{2}mv^2=0.5\,(20{,}000\text{ kg})\,(30.56\text{ m/s})^2=9.336\times10^{6}\text{ J}=\underline{9.34\times10^{6}\text{ J}}$$
$$\text{KE}_{\text{as}}=\frac{1}{2}mv^2=0.5\,(80.0\text{ kg})\,(7.638\times10^{3}\text{ m/s})^2=2.334\times10^{9}\text{ J}=\underline{2.33\times10^{9}\text{ J}}$$
$$\frac{\text{KE}_{\text{tr}}}{\text{KE}_{\text{as}}}=\frac{9.34\times10^{6}\text{ J}}{2.33\times10^{9}\text{ J}}=\frac{1}{\underline{250}}$$

10. *(a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.*

Solution

(a) $\text{KE}=\frac{1}{2}mv^2$, so for the same KE: $\frac{1}{2}m_{\text{elephant}}v^2_{\text{elephant}}=\frac{1}{2}m_{\text{sprinter}}v^2_{\text{sprinter}}$, and
$$v_{\text{elephant}}=\sqrt{\frac{m_{\text{sprinter}}}{m_{\text{elephant}}}v^2_{\text{sprinter}}}=\sqrt{\frac{65.0\text{ kg}}{3000\text{ kg}}(10.0\text{ m/s})^2}=\underline{1.47\text{ m/s}}$$
(b) If the elephant and the sprinter accelerate to a final velocity of 10.0 m/s, then the elephant would have a much larger kinetic energy than the sprinter. Therefore, the elephant clearly has burned more energy and requires a faster metabolic output to sustain that speed.

11. *Confirm the value given for the kinetic energy of an aircraft carrier in Table 7.1. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).*

Solution
$$90{,}000\text{ metric ton}\times\frac{2240\text{ lb}}{1\text{ metric ton}}\times\frac{0.4539\text{ kg}}{1\text{ lb}}=9.1506\times10^{7}\text{ kg;}$$
$$30\text{ knot}\times\frac{1.1516\text{ mi/h}}{1\text{ knot}}\times\frac{1609\text{ m}}{1\text{ mi}}\times\frac{1\text{ h}}{3600\text{ s}}=15.44\text{ m/s}$$
$$\text{KE}=\frac{1}{2}mv^2=0.5\,(9.1506\times10^{7}\text{ kg})\,(15.44\text{ m/s})^2=1.091\times10^{10}\text{ J}=\underline{1.1\times10^{10}\text{ J}}$$
(to two significant figures)

12. *(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).*

Solution

(a) $90.0\ \text{km/h}\times\dfrac{1000\ \text{m}}{1\ \text{km}}\times\dfrac{1\ \text{h}}{3600\ \text{s}}=25.0\ \text{m/s}$

$$W=Fd=\frac{1}{2}mv^2-\frac{1}{2}mv_0^{\ 2}\Rightarrow$$

$$F=\frac{-mv_0^{\ 2}}{2d}=\frac{-(950\ \text{kg})\,(25.0\ \text{m/s})^2}{2\times120\ \text{m}}=-2.474\times10^3\ \text{N}=\underline{-2470\ \text{N}}$$

$$F=\frac{-mv_0^{\ 2}}{2d}=\frac{-(950\ \text{kg})\,(25.0\ \text{m/s})^2}{2\times2.00\ \text{m}}=-1.484\times10^5\ \text{N}=\underline{-1.48\times10^5\ \text{N}}$$

(b) $\dfrac{F_\text{b}}{F_\text{a}}=\underline{60.0}$

13. *A car's bumper is designed to withstand a 4.0-km/h (1.12-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.12 m/s.*

Solution

Using the work energy theorem, $\text{net}\,W=\dfrac{1}{2}mv^2-\dfrac{1}{2}mv_0^{\ 2}=Fd\cos\theta$,

$$F=\frac{mv^2-mv_0^{\ 2}}{2d\cos\theta}=\frac{(900\ \text{kg})\,(0\ \text{m/s})^2-(900\ \text{kg})\,(1.12\ \text{m/s})^2}{2\,(0.200\ \text{m})\cos0^\circ}=\underline{-2.8\times10^3\ \text{N}}$$

The force is negative because the car is decelerating.

14. *Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?*

Solution

(a) $\text{net}\,W = Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2}$, since $v = 0$,

$F = \frac{-mv_0^{\ 2}}{2d}$ on the glove $\Rightarrow F = \frac{mv_0^{\ 2}}{2d}$ on the face, or

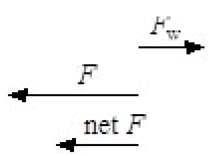
$$F = \frac{(7.00\ \text{kg})(10.0\ \text{m/s})^2}{2(0.0750\ \text{m})} = \underline{4.67\times10^3\ \text{N}}$$

(b) $F = \frac{mv^2}{2d} = \frac{(7.00\ \text{kg})(10.0\ \text{m/s})^2}{2(0.0200\ \text{m})} = \underline{1.75\times10^4\ \text{N}}$

(c) The force with the glove on is as if a 477 kg (1050 lb) mass were placed on the person's face. That definitely could do damage, even though it is less than the effective 1790 kg mass of the force with no gloves.

15. *Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.*

Solution



$$\text{net}\,F\times d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2} = \frac{1}{2}m(v^2 - v_0^{\ 2})$$

$$\text{net}\,F = \frac{m}{2d}(v^2 - v_0^{\ 2}) = \frac{60.0\,\text{kg}}{2\times25.0\,\text{m}}\left[(8.00\,\text{m/s})^2 - (2.00\,\text{m/s})^2\right] = 72.0\,\text{N}$$

$$\text{net}\,F = F - F_\text{W} \Rightarrow F = \text{net}\,F + F_\text{W} = 72.0\,\text{N} + 30.0\,\text{N} = \underline{102\ \text{N}}$$

7.3 GRAVITATIONAL POTENTIAL ENERGY

16. *A hydroelectric power facility (see Figure 7.35) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume* $50.0\ \text{km}^3$ *(* $\text{mass} = 5.00\times10^{13}\ \text{kg}$ *), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.*

Solution

(a) $\Delta\text{PE}_\text{g} = mgh$, so that $\Delta\text{PE}_\text{g} = (5.00\times10^{13}\ \text{kg})\,(9.80\ \text{m/s}^2)\,(40.0\ \text{m}) = \underline{1.96\times10^{16}\ \text{J}}$

(b) $\frac{E_\text{lake}}{E_\text{bomb}} = \frac{1.96\times10^{16}\ \text{J}}{3.8\times10^{16}\ \text{J}} = \underline{0.52}.$ The energy stored in the lake is approximately half that of a 9-megaton fusion bomb.