Solutions to problems assigned week of February 04, 2025.

13. ***A soccer player extends her lower leg in a kicking motion by exerting a force***

Solution

$$\text{net } \tau = rF = I\alpha \Rightarrow F = \frac{I\alpha}{r} = \frac{(0.750\ \text{kg}\cdot\text{m}^2)(30.0\ \text{rad/s}^2)}{0.0190\ \text{m}} = \underline{1.18\times10^3\ \text{N}}$$

15. ***Consider the 12.0 kg motorcycle wheel shown in Figure 10.36. Assume it to be approximately an annular ring …***

Solution

$$\tau = rF = 0.0500\ \text{m}\times 2200\ \text{N} = 110\ \text{N}\cdot\text{m}$$

$$\alpha = \frac{\tau}{I};\ I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$

$$I = 0.5(12.0\ \text{kg})\left[(0.280\ \text{m})^2 + (0.330\ \text{m})^2\right] = \underline{1.124\ \text{kg}\cdot\text{m}^2}$$

(a)
$$\alpha = \frac{110\ \text{N}\cdot\text{m}}{1.124\ \text{kg}\cdot\text{m}^2} = \underline{97.9\ \text{rad/s}^2}$$

(b) $a_t = \alpha R_2 = 97.9\ \text{rad/s}^2 \times 0.330\ \text{m} = \underline{32.3\ \text{m/s}^2}$

(c)
$$\omega = \omega_0 + \alpha t = \alpha t(\text{ since } \omega_0 = 0) \Rightarrow t = \frac{\omega}{\alpha} = \frac{80.0\ \text{rad/s}}{97.9\ \text{rad/s}^2} = \underline{0.817\ \text{s}}.$$

20. ***Unreasonable Results*** *An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel, ..*

Solution

Given: $M_{\text{car}} = 800\ \text{kg};\ M_{\text{fly}} = 20.0\ \text{kg};\ R_{\text{fly}} = 0.150\ \text{m};\ v = 30.0\ \text{m/s}$

$$\text{KE}_{\text{fly}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}M_{\text{fly}}R_{\text{fly}}^2\right)\omega^2$$

$$\text{KE}_{\text{t}} = \text{kinetic energy of car} = \frac{1}{2}\left(M_{\text{car}}v^2\right)$$

$$0.950\text{KE}_{\text{fly}} = \text{KE}_{\text{t}}$$

(a)
$$0.950\frac{1}{2}\left(\frac{1}{2}M_{\text{fly}}R_{\text{fly}}^2\right)\omega^2 = \frac{1}{2}M_{\text{car}}v^2$$

$$\omega = \left\{ \frac{M_{\text{car}} v^2}{(0.950)\left[(1/2) M_{\text{fly}} R_{\text{fly}}^2\right]} \right\}^{1/2}$$

$$= \left[\frac{(800 \text{ kg})(30.0 \text{ m/s})^2}{(0.950)(0.5)(20.0 \text{ kg})(0.150 \text{ m})^2} \right]^{1/2}$$

$$= 1835 \text{ rad/s} = \underline{1.84 \times 10^3 \text{ rad/s}} = 292 \text{ rev/s} = \underline{17{,}500 \text{ rpm}}$$

(b) This angular velocity is very high for a disk of this size and mass. The radial acceleration at the edge of the disk is > 50,000 gs.

(c) Flywheel mass and radius should both be much greater, allowing for a lower spin rate (angular velocity).

22. *What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?*

Solution $\text{PE}_{\text{grav}} = \text{KE}_{\text{trans}} + \text{KE}_{\text{rot}}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\left(\frac{v^2}{R^2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$mgh = mv^2 \Rightarrow gh = v^2$$

$$v = \sqrt{gh} = \left[\left(9.80 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)\right]^{1/2} = \underline{7.00 \text{ m/s}}$$

27. *A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg. ..*

Solution $(90.0\%)\text{KE}_{\text{rot}} = \text{KE}_{\text{trans}}$ so that $(0.900)\frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2$.

Let M be the mass of the bus and m be the mass of the flywheel.

$$I\omega^2 = \frac{Mv^2}{0.900}, \text{ and since the flywheel is a disc :}$$

$$I = \frac{1}{2}mr^2 \Rightarrow \frac{1}{2}mr^2\omega^2 = \frac{Mv^2}{0.900}$$

(a) $$\omega = \sqrt{\frac{2Mv^2}{0.900mr^2}} = \left[\frac{2\left(10{,}000 \text{ kg}\right)\left(20.0 \text{ m/s}\right)^2}{\left(0.900\right)\left(1500 \text{ kg}\right)\left(0.600 \text{ m}\right)^2}\right]^{1/2} = \underline{128 \text{ rad/s}}$$

$$\mathrm{KE}_{\text{trans}} = \mathrm{PE'}_{\text{grav}} + \mathrm{KE'}_{\text{trans}} \text{ so that } (0.900)\frac{1}{2}I\omega^2 = Mgh' + \frac{1}{2}Mv'^2,$$

(b) $$\text{or } (0.900)\frac{1}{4}mr^2\omega^2 = Mgh' + \frac{1}{2}Mv'^2, \text{ so that}$$

$$h' = \frac{0.900\ \mathrm{mr}^2\omega^2}{4Mg} - \frac{Mv'^2}{2g} = \frac{0.900mr^2\omega^2 - 2Mv'^2}{4Mg}$$

$$h' = \frac{\left[(0.900)(1500\ \text{kg})(0.600\ \text{m})^2(128.3\ \text{rad/s})^2 - 2(10{,}000\ \text{kg})(3.00\ \text{m/s})^2\right]}{4(10{,}000\ \text{kg})(9.80\ \text{m/s}^2)} = \underline{19.9\ \text{m}}$$

32. What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest, ..

Solution $$\mathrm{PE}_{\text{grav}} = \mathrm{KE'}_{\text{trans}} + \mathrm{KE'}_{\text{rot}}$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$I = \frac{2\left[Mgh - (1/2)Mv^2\right]}{\omega^2} = \frac{2M}{\omega^2}\left(gh - \frac{1}{2}v^2\right) = \frac{2MR^2\left[gh - (1/2v^2)\right]}{v^2}$$

$$I = 2MR^2\,\frac{\left[(9.80\ \text{m/s}^2)(2.00\ \text{m}) - (0.5)(6.00\ \text{m/s})^2\right]}{(6.00\ \text{m/s})^2}$$

$$= 2MR^2\left[\frac{1.60\ \text{m/s}^2}{36.0\ \text{m/s}^2}\right] = \underline{\left(\frac{4}{45}\right)MR^2} \text{ or } \underline{0.0889MR^2}$$

39. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s..

Solution $$L = L' \Rightarrow I\omega = I'\omega'$$

$$I' = I + I_c, \text{ where } I_c = mr^2$$

$$\omega' = \frac{I}{I'}\omega = \frac{I}{I + I_c}\omega$$

$$= \frac{(1/2)Mr^2}{(1/2)(M + 2m)r^2}\omega = \left(\frac{M}{M + 2m}\right)\omega = \left(\frac{120\ \text{kg}}{164\ \text{kg}}\right)(0.500\ \text{rev/s})$$

$$\omega' = 0.366\ \text{rev/s} = \underline{2.30\ \text{rad/s}}$$