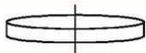
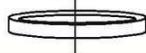
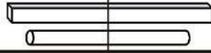
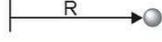


Modeling Rotational Inertia

Introduction:

In systems undergoing linear motion, we must consider the objects 'resistance' to acceleration which is defined as the Inertia of the object. In linear motion, the inertia is simply determined by measuring the mass (in Kg). In rotational systems, we also consider the 'resistance' to rotational acceleration and also refer to that as Rotational Inertia but in the case of a rotating system, we must also consider the distribution of mass.

And for systems that include several parts, the rotational inertia can be modeled as simply the 'sum' of the inertias of the respective parts.

Object	Drawing	Moment of Inertia
Disk (rotated about center)		$\frac{1}{2}MR^2$
Ring (rotated about center)		MR^2
Rod or plank (rotated about center)		$\frac{1}{12}ML^2$
Rod or plank (rotated about end)		$\frac{1}{3}ML^2$
Sphere		$\frac{2}{5}MR^2$
Satellite		MR^2

For this lab experiment, students will construct a several-part rotational system, apply a torque to it and then measure directly, the rotational acceleration. By knowing the applied torque we can then determine what the actual Rotational Inertia must be.

Part 1: Modeling the Rotational Inertia of the system.

Modeling of the rotational inertia consists of nothing more than identifying a collection of pre-defined shapes that best fit the overall design of the system. For this lab, we have a rectangular plate (being rotated about its center) and two, smaller mass-assemblies, being rotated a 'distance out' from the axis (referred to as a Satellite in the above Table). By determining the mass of these three pieces and measuring their respective dimensions, we can calculate the three separate Moments of Inertia which are then added to determine the Total Rotational Inertia. (note: the units will be $\text{Kg}\cdot\text{m}^2$).

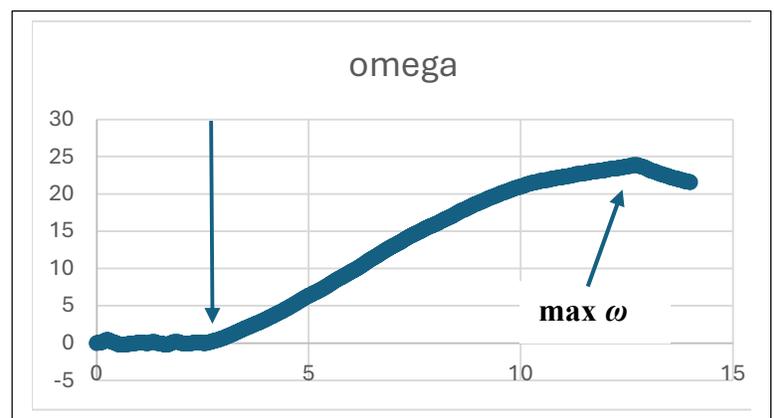
Part 2; Using laws of conservation of energy to determine Rotational Inertia.

We have already learned that Work done on a system is the product of the applied force and the distance moved and that due to conservation of energy, that that work is often converted into motion resulting in kinetic energy.

$$\text{Work} = \text{Force} \times \text{distance} \implies \text{K.E.} = \frac{1}{2} I \omega^2.$$

(for a rotating system).

Therefore, if we can determine the work that we did, then we can know the kinetic energy of the system. And if we know the 'max ω ' of that system, we can solve for the Moment of Inertia, I.



Rotation starts at 2.58 sec and 'peaks' at 12.65 sec.

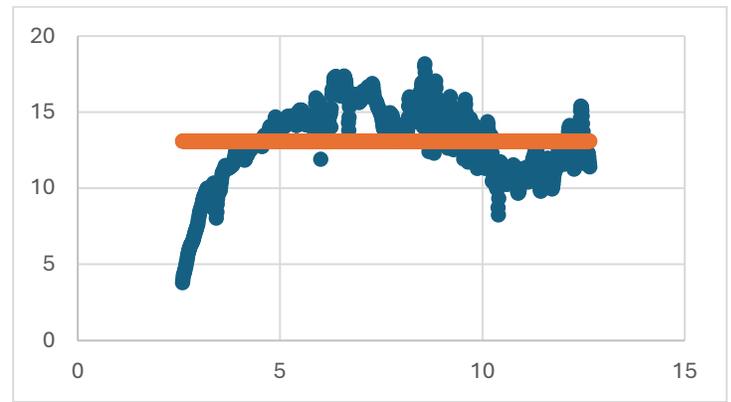
Determining the Work done however, requires several steps.

$$\text{Work} = \text{Force} \times \text{distance}$$

Though we have force data for the entire run, we are only interested in force being applied WHILE the system was being accelerated..

By examining the rotational velocity graph, one can determine the actual ‘start’ of the acceleration and the ‘end’ of the experiment (where the string released from the shaft).

By plotting force data for THIS time interval, we can determine the ‘average’ force applied over that time-span.



Average applied force was 13.1 newtons.

The Distance is equal to the length of string that was drawn out but... Noting that the string slips as it gets towards the end, we don’t use the measured length of string, but instead use the measured angular distance (radians of rotation since this can be converted into linear distance through the identity $\Delta x = \Delta\theta \times r$ radius).

Unfortunately, the transponder doesn’t give us radians directly, so we must determine the number of rotations through integration (the area under the angular velocity curve).

To determine the ‘area under the curve’.. write a function which multiplies the magnitude of the of velocity by the time interval between each measurement. (i.e., $\text{area} = \Delta\theta = \omega \times \Delta T$... eg.. if the frequency of measurement was 100 Hz, the time interval will be 0.01 sec. If we do this for every measurement and then add them up, we can approximate the TOTAL angular displacement that took place while the string was being pulled. My multiplying this value by the radius of the shaft, we can know that total linear distance that the string went through while pulling.

Part 3: Using Impulse and rotational momentum to determine the Rotational Inertia of our system.

In linear systems, $\text{Force} = \Delta P / \Delta T$.

In rotating systems: $\text{Torque} = \Delta L / \Delta T$ (where $\Delta L = I \Delta \omega$)

Since $\text{Torque} = \text{Force} \times \text{radial distance}$, one can simply multiply the Force data set by the radius of the shaft to create a Torque (as a function of time) data set. And by multiplying each of those Torque values by the time interval, (0.01 seconds in this example) we can generate a data set which will equal the incremental impulses the system received. (note: $\text{Impulse} = \text{Torque} \times \Delta T = I \Delta \omega$).

Finally, by dividing the total Impulse by the ‘change in Omega’, we can determine what the Rotational Inertia must have been. . (note: The Change in Omega is simply equal to the ‘max’ omega since the system started off at rest.

Part 4: Compare the three values for I. (modeled with Tables of Inertia compared to the two, measured values).