

20. Unreasonable Results An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel,

Solution Given: $M_{\text{car}} = 800 \text{ kg}$; $M_{\text{fly}} = 20.0 \text{ kg}$; $R_{\text{fly}} = 0.150 \text{ m}$; $v = 30.0 \text{ m/s}$

$$\text{KE}_{\text{fly}} = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} M_{\text{fly}} R_{\text{fly}}^2 \right) \omega^2$$

$$\text{KE}_t = \text{kinetic energy of car} = \frac{1}{2} (M_{\text{car}} v^2)$$

$$0.950 \text{KE}_{\text{fly}} = \text{KE}_t$$

$$(a) \quad 0.950 \frac{1}{2} \left(\frac{1}{2} M_{\text{fly}} R_{\text{fly}}^2 \right) \omega^2 = \frac{1}{2} M_{\text{car}} v^2$$

$$\begin{aligned} \omega &= \left\{ \frac{M_{\text{car}} v^2}{(0.950) \left[\frac{1}{2} M_{\text{fly}} R_{\text{fly}}^2 \right]} \right\}^{1/2} \\ &= \left[\frac{(800 \text{ kg})(30.0 \text{ m/s})^2}{(0.950)(0.5)(20.0 \text{ kg})(0.150 \text{ m})^2} \right]^{1/2} \\ &= 1835 \text{ rad/s} = \underline{1.84 \times 10^3 \text{ rad/s}} = 292 \text{ rev/s} = \underline{17,500 \text{ rpm}} \end{aligned}$$

(b) This angular velocity is very high for a disk of this size and mass. The radial acceleration at the edge of the disk is $> 50,000 \text{ gs}$.

(c) Flywheel mass and radius should both be much greater, allowing for a lower spin rate (angular velocity).

22. What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?

Solution $\text{PE}_{\text{grav}} = \text{KE}_{\text{trans}} + \text{KE}_{\text{rot}}$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} mv^2 + \frac{1}{2} mR^2 \left(\frac{v^2}{R^2} \right) = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$mgh = mv^2 \Rightarrow gh = v^2$$

$$v = \sqrt{gh} = \left[(9.80 \text{ m/s}^2)(5.00 \text{ m}) \right]^{1/2} = \underline{7.00 \text{ m/s}}$$

27. A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg.

Solution $(90.0\%)KE_{\text{rot}} = KE_{\text{trans}}$ so that $(0.900)\frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2$.

Let M be the mass of the bus and m be the mass of the flywheel.

$$I\omega^2 = \frac{Mv^2}{0.900}, \text{ and since the flywheel is a disc :}$$

$$I = \frac{1}{2}mr^2 \Rightarrow \frac{1}{2}mr^2\omega^2 = \frac{Mv^2}{0.900}$$

$$(a) \quad \omega = \sqrt{\frac{2Mv^2}{0.900mr^2}} = \left[\frac{2(10,000 \text{ kg})(20.0 \text{ m/s})^2}{(0.900)(1500 \text{ kg})(0.600 \text{ m})^2} \right]^{1/2} = \underline{128 \text{ rad/s}}$$

$$KE_{\text{trans}} = PE'_{\text{grav}} + KE'_{\text{trans}} \text{ so that } (0.900)\frac{1}{2}I\omega^2 = Mgh' + \frac{1}{2}Mv'^2,$$

$$(b) \quad \text{or } (0.900)\frac{1}{4}mr^2\omega^2 = Mgh' + \frac{1}{2}Mv'^2, \text{ so that}$$

$$h' = \frac{0.900mr^2\omega^2}{4Mg} - \frac{Mv'^2}{2g} = \frac{0.900mr^2\omega^2 - 2Mv'^2}{4Mg}$$

$$h' = \frac{[(0.900)(1500 \text{ kg})(0.600 \text{ m})^2(128.3 \text{ rad/s})^2 - 2(10,000 \text{ kg})(3.00 \text{ m/s})^2]}{4(10,000 \text{ kg})(9.80 \text{ m/s}^2)} = \underline{19.9 \text{ m}}$$

32. What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest...

Solution $PE_{\text{grav}} = KE'_{\text{trans}} + KE'_{\text{rot}}$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$I = \frac{2[Mgh - (1/2)Mv^2]}{\omega^2} = \frac{2M}{\omega^2} \left(gh - \frac{1}{2}v^2 \right) = \frac{2MR^2 [gh - (1/2)v^2]}{v^2}$$

$$I = 2MR^2 \frac{[(9.80 \text{ m/s}^2)(2.00 \text{ m}) - (0.5)(6.00 \text{ m/s})^2]}{(6.00 \text{ m/s})^2}$$

$$= 2MR^2 \left[\frac{1.60 \text{ m/s}^2}{36.0 \text{ m/s}^2} \right] = \left(\frac{4}{45} \right) MR^2 \text{ or } \underline{0.0889MR^2}$$

39. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.

Solution

$$L = L' \Rightarrow I\omega = I'\omega'$$

$$I' = I + I_c, \text{ where } I_c = mr^2$$

$$\omega' = \frac{I}{I'}\omega = \frac{I}{I + I_c}\omega$$

$$= \frac{(1/2)Mr^2}{(1/2)(M + 2m)r^2}\omega = \left(\frac{M}{M + 2m}\right)\omega = \left(\frac{120 \text{ kg}}{164 \text{ kg}}\right)(0.500 \text{ rev/s})$$

$$\omega' = 0.366 \text{ rev/s} = \underline{2.30 \text{ rad/s}}$$